Bananas at $0.63/lb., the Speed of Light, and Quantum Mechanics

OPCUG & PATACS

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About the title…

Bananas at $0.63/lb.,
the Speed of Light, and
Quantum Mechanics

Yes, these things are related!

Heads Up!

- Some concepts are complex, unintuitive and quite mathematical
- Simplifications have been made
- Resulting in:
  - ✓ easier understanding
  - ✓ whole story isn't told
  - ✓ inaccuracies introduced

On the other hand…

- Rigorous exposition would result in:
  - ✓ boredom—too much talking
  - ✓ minds drifting
  - ✓ little understanding

You can't have it all!

A Little Background…

- March 21, 2020—Presentation "SI* Units—Who Cares?"
- Importance of standards
- Systems of measurement
  - ✓ International System of Units (SI)—modern metric system
  - ✓ U.S. Customary Units
  - ✓ Imperial System of Units
- Overview of the seven basic SI units (standards)
- Bill Walsh's surprised about the new standard kg

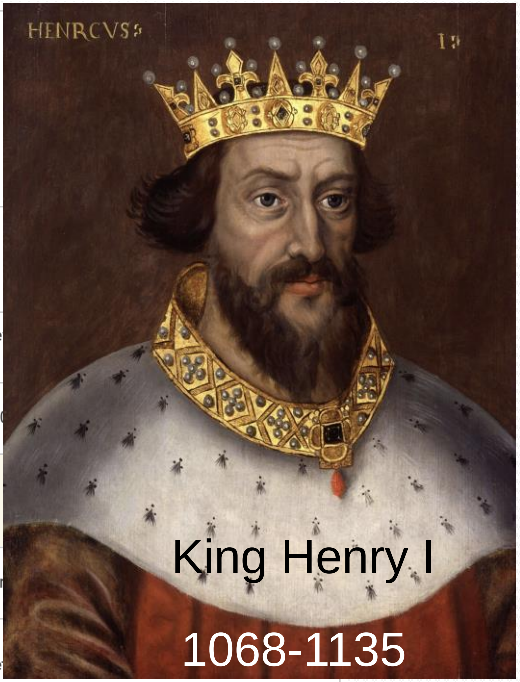
* SI = "Système International", i.e., the metric system

Today's Focus…

Some background information

The **Many** Units of Length

Link (Gunter's link)	Length	l., li. or lnk.	Imperial / USCS	1 link = 0.01 chain = 0.04 rod = 0.66 foot = 0.22 yard = 7.92 inches = 0.201168 meters. Gunter's chain, designed by Edmund Gunter in 1620 in England, was a metal chain made up of 100 links. Each link inches. The link was widely used in surveying in English speaking countries until the 20th century. • Twenty-five links make a rod • One hundred links make a chain • One thousand links make a furlong • Eight thousand links make a mile.
Foot	Length	ft	Imperial / USCS	12 inches (or 30.48 cm or 304.8 mm). • The **international foot** is a exactly 0.3048 meters • **US Survey Foot** is a fraction: 1200/3937 meters, a difference of one one-hundredth of a foot per mile
Yard	Length	yd	Imperial / USCS	3 feet or 36 inches (or 91.44 cm or 914.4 mm). The international yard is defined as exactly 0.9144 metres so it is e units of measurement.
Rod/Perch / Pole/Lug	Length		Imperial / USCS	1 rod is 16 $^1/_2$ US survey feet = 5.0292 m. To convert US feet to international feet multiply by 1.000002000004000
Chain	Length		Imperial / USCS	1 chain = 66 feet (22 yards) = 20.1168 meters. 1 chain = 4 rods or 100 links. 10 chains = 1 furlong. 80 chains = 1 international/statute mile. The distance between the stups on a cricket pitch is still defined as one chain.
Furlong	Length		Imperial / USCS	1 furlong is $^1/_8$ of a mile. One furlong = 220 yards = 201.1680 meters. Race lengths in furlongs are still used in mar
Mile	Length	mi or m or ml	Imperial / USCS	1,760 yards or 5,280 feet. In 1959, by international agreement, one mile was standardized as exactly 1,609.344 me
Nautical Mile	Length	sm	Imperial	exactly 1,852 meters (or about 6,076 feet). Shot = 15 fathoms or 90 feet
Astronomical Unit	Length	AE	Metric	149,597,870,700 m (or 1.49598 x 10^{11} m).
Light Year	Length	lj	Metric	9.4607 x 10^{15} m (or 9.4607 Pm)
Parsec	Length	pc	Metric	3.0857 x 10^{16} m.



See ← Hypertext links to more information (94 in total)

Many Values for the Foot*

Modern Country	Location	Metric Equivalent (mm)
Austria	Tyrol	334.12
Belgium	Bruges	274.3
Belgium	Ypres	273.8
France	Bordeaux	357.214
France	Strasbourg	294.95
Germany	Berlin	309.6
Germany	Bremen	289.35
Italy	Rome	297.896
Netherlands	Amsterdam	283.133
Norway	—	313.75
Scotland	—	305.287
Sweden	—	296.9

} 71 km (44 mi) apart

* All these countries now use the meter.

A Tale of Two Feet in the U.S.

- International foot of 1959, I_{foot}
- U.S. Survey foot of 1893*, S_{foot}

$I_{foot} = 0.999998\ S_{foot}$

* S_{foot} to be avoided effective January 1, 2023, according to NIST, see

Miles/Hour

Furlongs/Minute

↑ Used in horse racing

Engines rated
in horsepower

Metrification of the U.S.

- U.S pound = 0.45359237 kg
- International (U.S.) foot = 0.3048 m
- Temperature—Absolute zero*
  - ✓ 0 K
  - ✓ − 273.15°C
  - ✓ − 459.67°F

LED light bulb
2700K soft white

$$°C = (°F - 32) \times 5 / 9$$

Celsius to kelvin

$$K = °C + 273.15$$

$$°F = (°C \times 9 / 5) + 32$$

Kelvin to Celsius

$$°C = K - 273.15$$

- The U.S. uses the metric (IS) units for: the second, ampere, mole and candela—but not the meter, kilogram or °C

* The state at which the enthalpy and entropy of a cooled ideal gas reach their minimum value

Lowest temp reached 3.8×10^{-11} K

International System of Units

(aka the modern metric system)

SI base units

Symbol	Name	Quantity
s	second	time
m	metre	length
kg	kilogram	mass See
A	ampere	electric current
K	kelvin	thermodynamic temperature
mol	mole	amount of substance
cd	candela	luminous intensity

*

International System of Units

(aka the modern metric system)

Spoiler alert

SI defining constants

	Symbol	Defining constant	Exact value
time, s	$\Delta\nu_{Cs}$	hyperfine transition frequency of Cs ✓	9 192 631 770 Hz
length, m	c	speed of light ✓	299 792 458 m/s
* mass, kg	h	Planck constant ✓	$6.626\,070\,15 \times 10^{-34}$ J⋅s
electric current, A	e	elementary charge	$1.602\,176\,634 \times 10^{-19}$ C
(temp K)	k	Boltzmann constant ✓	$1.380\,649 \times 10^{-23}$ J/K
amount of substance, mol	N_A	Avogadro constant ✓	$6.022\,140\,76 \times 10^{23}$ mol^{-1}
luminous intensity, cd	K_{cd}	luminous efficacy of 540 THz radiation	683 lm/W

Measuring the Speed of Light

History of measurements of *c* (in km/s)

<1638	Galileo, covered lanterns	inconclusive[118][119][120]:1252[Note 15]	
<1667	Accademia del Cimento, covered lanterns	inconclusive[120]:1253[121]	
1675	Rømer and Huygens, moons of Jupiter	220 000[94][122]	–27% error
1729	James Bradley, aberration of light	301 000[105]	+0.40% error
1849	Hippolyte Fizeau, toothed wheel	315 000[105]	+5.1% error
1862	Léon Foucault, rotating mirror	298 000 ±500[105]	–0.60% error
1907	Rosa and Dorsey, EM constants	299 710 ±30[108][109]	–280 ppm error
1926	Albert A. Michelson, rotating mirror	299 796 ±4[123]	+12 ppm error
1950	Essen and Gordon-Smith, cavity resonator	299 792.5 ±3.0[111]	+0.14 ppm error
1958	K.D. Froome, radio interferometry	299 792.50 ±0.10[115]	+0.14 ppm error
1972	Evenson *et al.*, laser interferometry	299 792.4562 ±0.0011[117]	–0.006 ppm error
1983	17th CGPM, definition of the metre	299 792.458 (exact)[92]	exact, as defined

From Wikipedia see

CGPM = Conférence Générale des Poids et Mesures
General Conference on Weights and Measures

Improved Methods of Measurement

- What happens if measurement of the standard- dependent object changes (like the speed of light)?
  - ✓ Most likely outcome — result is a better value, i.e. less uncertainty and greater accuracy
  - ✓ A decision could be made to adopt the new, more accurate value for a standard**, but**
- **The bases on which the standards are derived do not change, it's the measurement that changes (an improvement)**

The Kilogram (kg)

History of the Kilogram (kg)

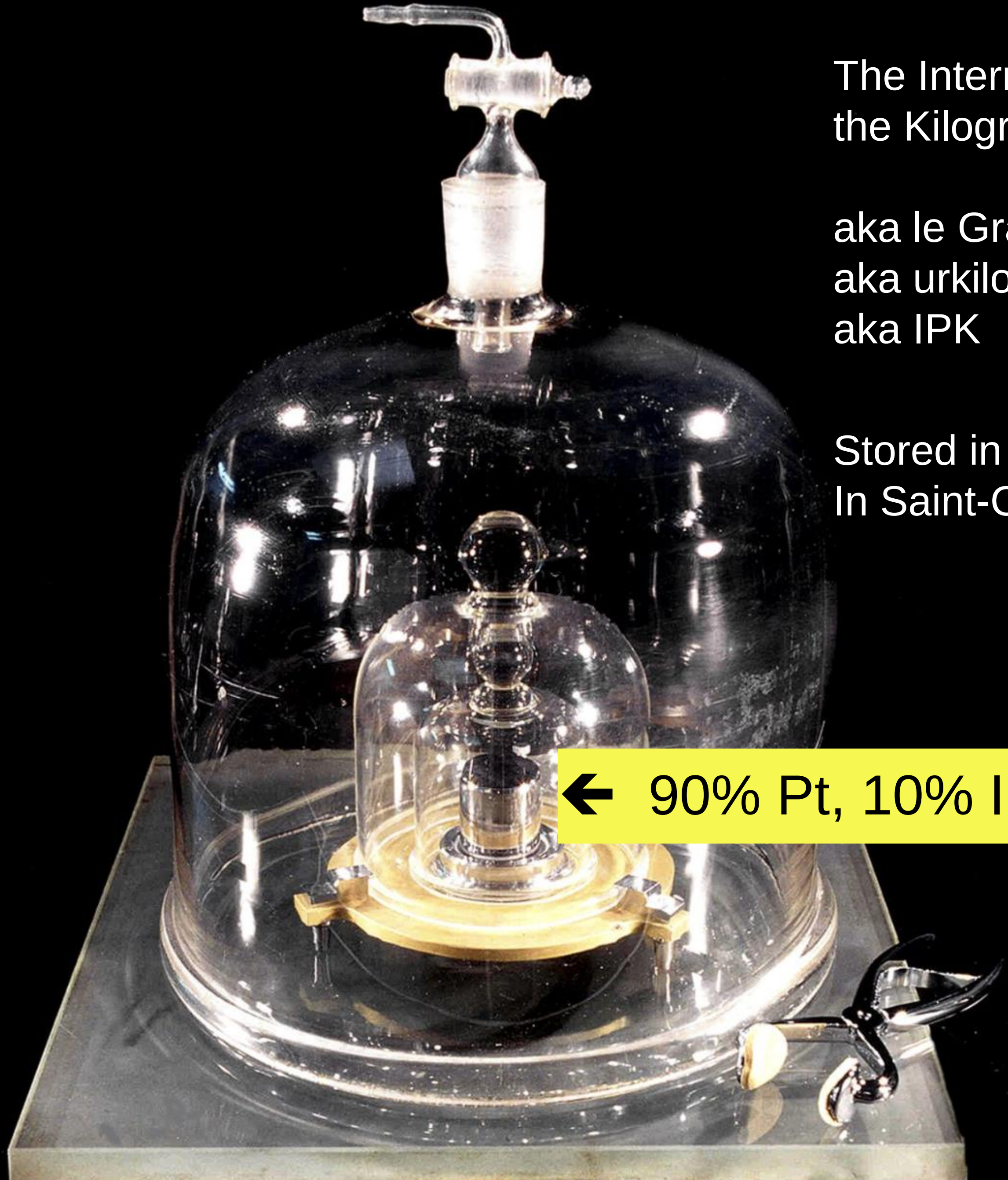
- 1793—The grave (precursor to the kg) mass of 1 liter of water at 0°C (18,841 grains[1])
- 1799—The prototype Kilogramme des Archives the mass of 1 dm^3 of water at its maximum density (about 4°C)
- 1889—The IPK (International Prototype of the Kilogram) established and in use **until November 16, 2018**

[1] 1 grain = mass of a single grain of barley

The International Prototype of the Kilogram

aka le Grand K
aka urkilogram
aka IPK

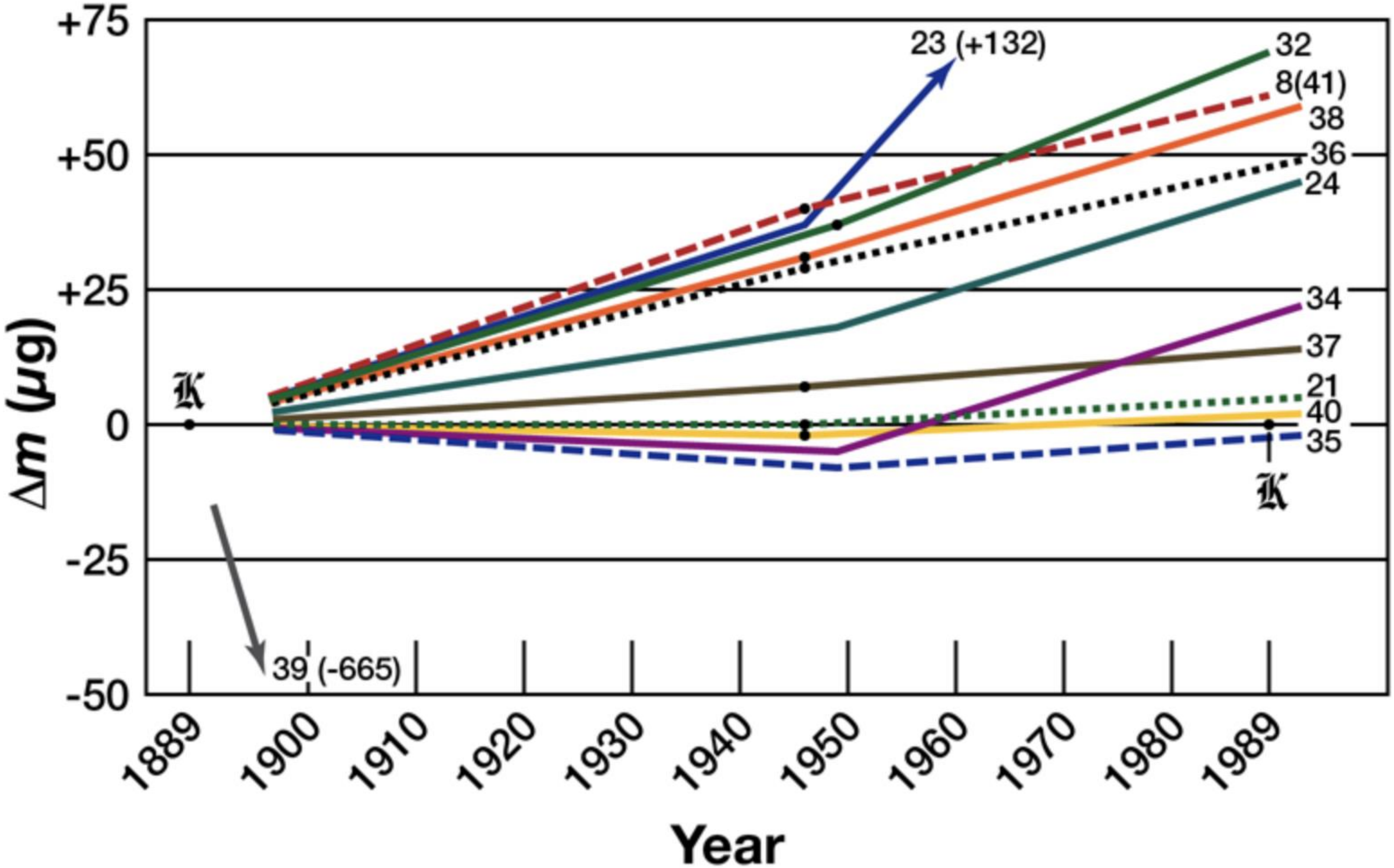
Stored in the Pavillon de Breteuil
In Saint-Cloud, France



← 90% Pt, 10% Ir

Why Change the kg Standard?

- Objective to create a precise kg of unchanging value
- Comparisons were made of the 1889 IPK, 6 sister copies and the national prototypes…
  - ✓ Pooled standard deviation of repeated weighings of the prototypes was ±0.4 μg (1 μg = 0.000001 g)
  - ✓ Cleaning and washing removed a mass of ~15 ± 2 μg
- See for more information



Mass drift over time of national prototypes K21–K40, plus two of the IPK's sister copies: K32 and K8(41).[Note 2] All mass changes are relative to the IPK. The initial 1889 starting-value offsets relative to the IPK have been nulled.[17] The above are all *relative* measurements; no historical mass-measurement data is available to determine which of the prototypes has been most stable relative to an invariant of nature. There is the distinct possibility that *all* the prototypes gained mass over 100 years and that K21, K35, K40, and the IPK simply *gained less* than the others.

} Note

More essential background information

Ludwig Boltzmann
(1844-1906)

How can I explain this?

Boltzmann's Constant (k_B) relates the average relative kinetic energy of particles in a gas with the thermodynamic temperature of the gas (K)

$$k_B = 1.380649 \times 10^{-23}\ \text{J/K}$$

One J = a Watt-second; 1/3600 Wh

For information on the Boltzmann Constant, see

Boltzmann's Constant Explained

$PV = nRT$

P is pressure in Pascals (Pa) [atm]

V is volume in m^3 [L]

n is amount of material in moles [mol]

R is the gas constant 8.31446 J/mol-K [0.0820574 L-atm/mol-°K]

$PV = Nk_BT$

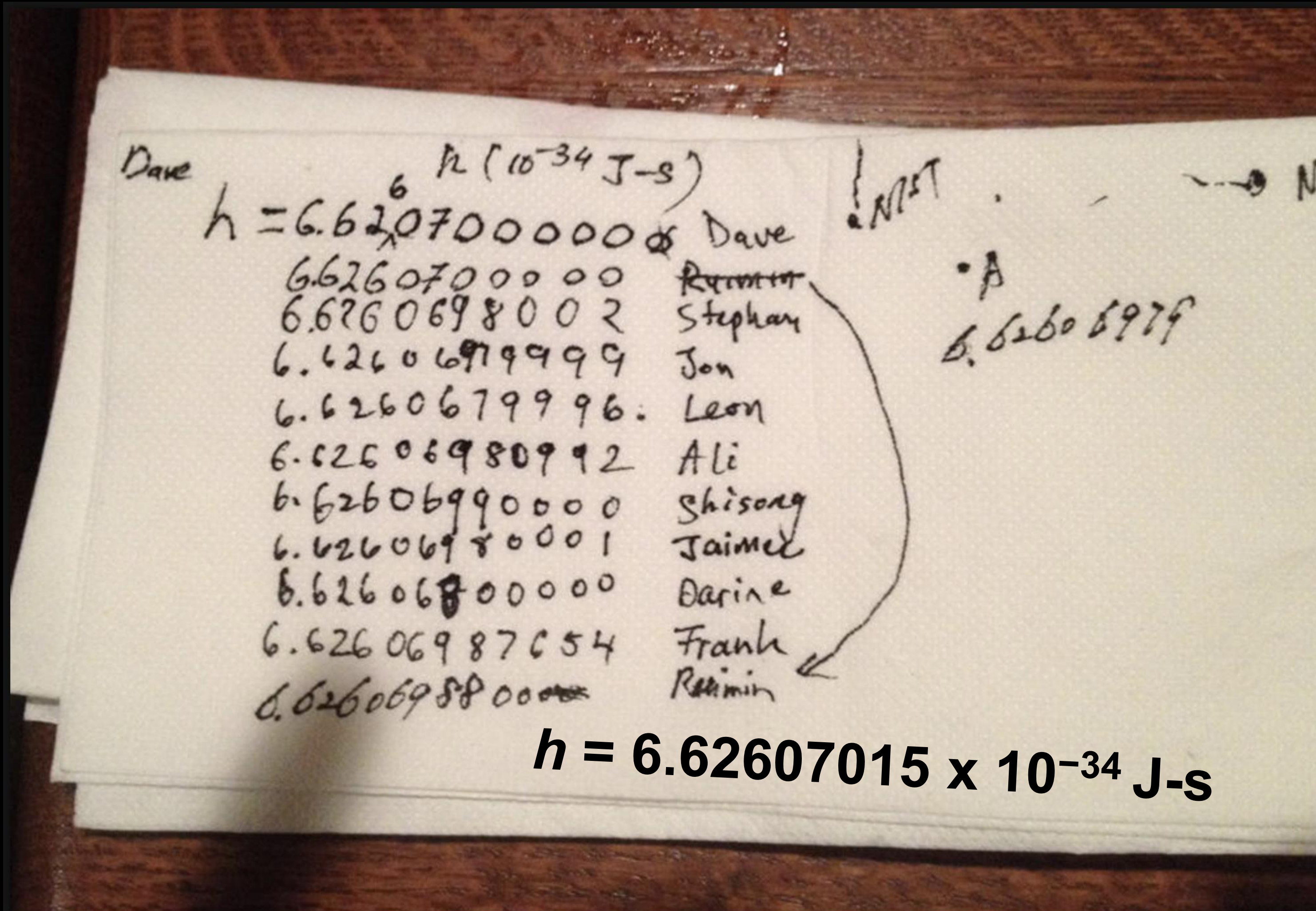
P is pressure in Pascals (Pa)

V is volume in m^3

N is number of atoms or molecules

k_B Boltzmann's Constant 1.380649 x 10^{-23} J/K

And Now, Planck's Constant



In December 2013, NIST group members wrote their predictions on the value of Planck's constant they would measure on their NIST-4 Kibble balance. Shisong Li, a guest researcher from Tsinghua University in China, came closest. His prediction differed by only about 5 parts per billion from the measured result. 2 of 3

We're Not Going To Talk about…

$$\int_{\lambda_1}^{\lambda_2} B_\lambda(\lambda, T) d\lambda = \int_{\nu(\lambda_2)}^{\nu(\lambda_1)} B(\nu, T) d\nu =$$

$$\int_{\lambda_2}^{\lambda_1} B(\nu, T)\frac{d\nu}{d\lambda} d\lambda = \int_{\lambda_1}^{\lambda_2} -B(\nu, T)\frac{d\nu}{d\lambda} d\lambda$$

$$B_\lambda(\lambda, T) = -B(\nu, T)\frac{d\nu}{d\lambda}$$

Using $c = \lambda\nu$, we see that

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

Max Planck
(1858-1947)

Planck's Constant

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_\mathrm{B}T}\right) - 1}$$

B = spectral radiance of a body (energy)

ν = frequency

T = absolute temperature

k_B = Boltzmann constant

h = Planck constant

c = speed of light in the medium

For more information see

Getting to E = *h*f

Energy of Planck mass.

$$E = m_P c^2$$

No further discussion

Trust me!!

Transverse energy for photon on one side of atom is half. In-wave and out-wave geometries applied (α_1) and spin of particle (α_2).

$$E_t = \frac{1}{2} m_P c^2 \left(\frac{\alpha_{1B}}{\alpha_{1A}}\right)(\alpha_{2B})$$

In-wave geometry. Refer to Spacetime geometry page.

$$\alpha_{1A} = \frac{q_P^2}{4\pi l_P^2}$$

Out-wave geometry. Refer to Spacetime geometry.

$$\alpha_{1B} = \frac{q_P^2}{4\pi r^2}$$

Particle spin geometry (α_2) for electron is the fine structure constant.

$$\alpha_{2B} = \frac{r_e^2}{4\pi(\pi r_e)^2 + (\pi(r_e)(\pi r_e) + \pi r_e^2)} = \frac{1}{(4\pi^3 + \pi^2 + \pi)} = \alpha_e$$

Getting to E = *h*f

No discussion
Trust me!!

Insert geometries from Eqs. 1.3.3 to 1.3.5 into Eq. 1.3.2.

$$E_t = \frac{1}{2} m_P c^2 \left(\frac{\frac{q_P^2}{4\pi r^2}}{\frac{q_P^2}{4\pi l_P^2}} \right) (\alpha_e)$$

Rerrange terms in Eq. 1.3.6.

$$E_t = \frac{1}{2} m_P c \left(\frac{\frac{q_P^2}{r}}{\frac{q_P^2}{4\pi l_P^2}} \right) \left(\frac{\alpha_e c}{4\pi r} \right)$$

Radius (r) of term *on the left* is Planck length, consistent with derivation of forces.

$$r = l_P$$

Substitute for r.

$$E_t = \frac{1}{2} m_P c \left(\frac{\frac{q_P^2}{l_P}}{\frac{q_P^2}{4\pi l_P^2}} \right) \left(\frac{\alpha_e c}{4\pi r} \right)$$

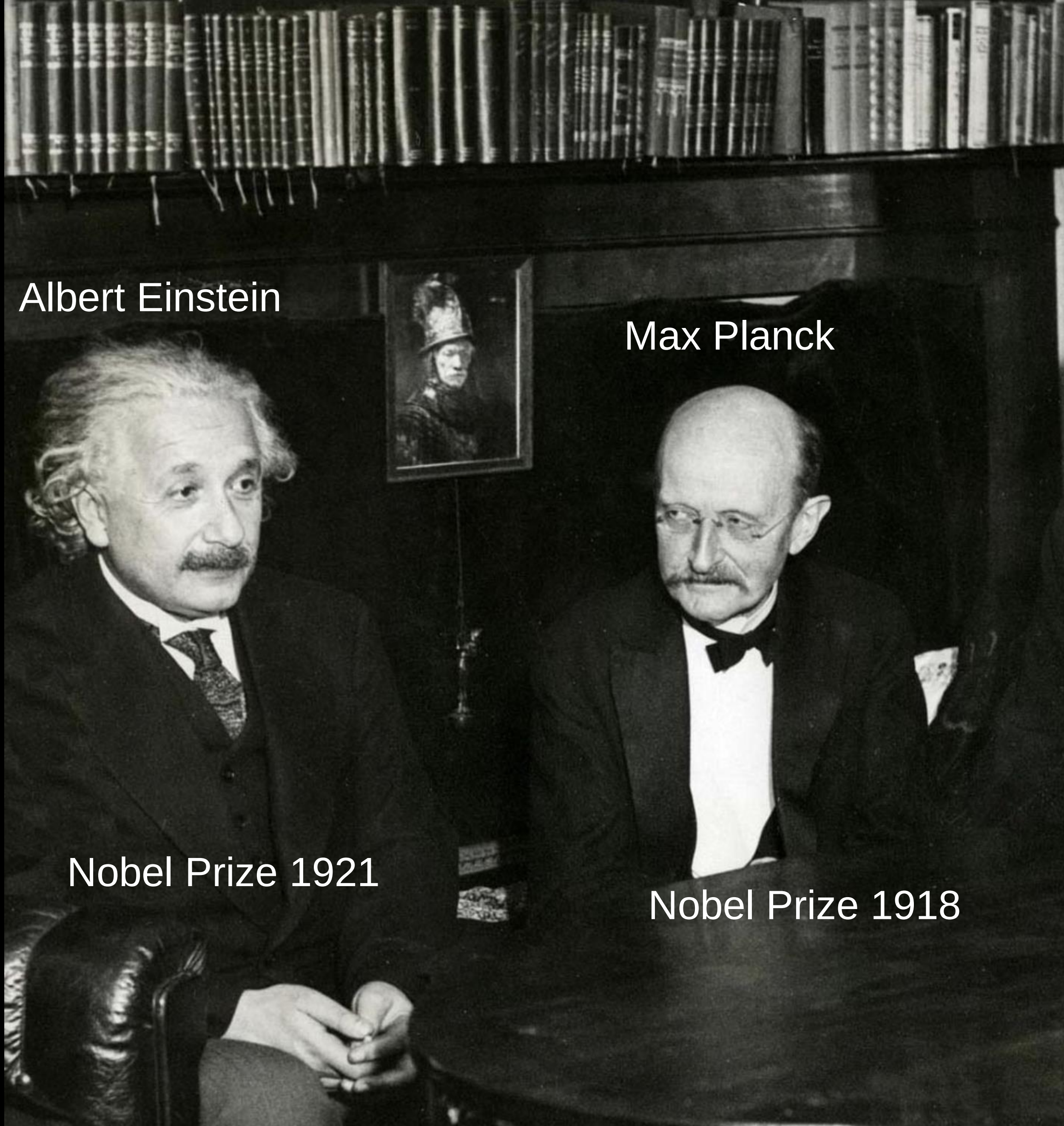
Getting to E = *h*f

No discussion
Trust me!!

Simplify equation.	$E_t = 2\pi l_P m_P c \left(\frac{\alpha_e c}{4\pi r}\right)$
Terms on the left of Eq. 1.3.10 is the Planck constant (h).	$h = 2\pi l_P m_P c$
The terms on the right of Eq. 1.3.10 are frequency. A symbol for constructive wave interference is added for multiple particles (Δ).	$f = \frac{\Delta\alpha_e c}{4\pi r}$
Subtitute Eq. 1.3.11 and 1.3.12 into Eq. 1.3.10.	$E_t = hf$

Planck's Constant

- The relationship between the energy of a photon and its frequency, i.e. **E = *h*f** (remember this equation)
- $h = 6.62607015 \times 10^{-34}$ J-Hz^{-1}
- *A fundamental constant of foundational importance in quantum mechanics*
- E = *h*f is also called the Planck-Einstein equation
- For more information see



Max von Laue

Theodore H. von Laue

Einstein and Planck at a dinner held by Max von Laue November 11, 1931

The Mole—Avogadro's Number

Avogadro's Number—the Mole

- $N_A = 6.02214076 \times 10^{23}$ mol^{-1} as defined in 2019
- The number of atoms or molecules in one mole is dependent on a substance's atomic or molecular weight, so…
- In 12.00000 g of ^{12}C there are $6.02214076 \times 10^{23}$ atoms of carbon
- In 18.00988 g of H_2O… same number
- In 27.9769265325 g of Si… the same
- See

Amedeo Avogadro

Interrelationships

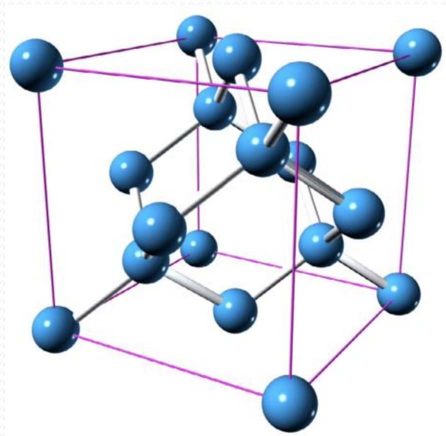
- Boltzmann's constant
- Planck's constant
- Avogadro's number

} ← these are interrelated

Sneaking up on the new kilogram…

Defining the kg By Number of Si Atoms

- Volume of a sphere $V = \frac{4}{3}\pi r^3$
- Knowing the size of the ^{28}Si unit cell (0.543 nm)



🡸 18 atoms/cell

- The kg <u>could be defined</u> as the mass of 2.15253873 x 10^{25} atoms of ^{28}Si
- For more information, see

Good, but not good enough by itself

It gets better!

* Useful in establishing Avogadro’s number

Now Comes Quantum Mechanics

$$i\hbar \frac{d}{dt}\psi(t) = H\psi(t)$$

Schrödinger equation

$$\sigma_A \sigma_B \geq \frac{1}{2} \left| \langle [A, B] \rangle \right|$$

Heisenberg's Uncertainty Principle

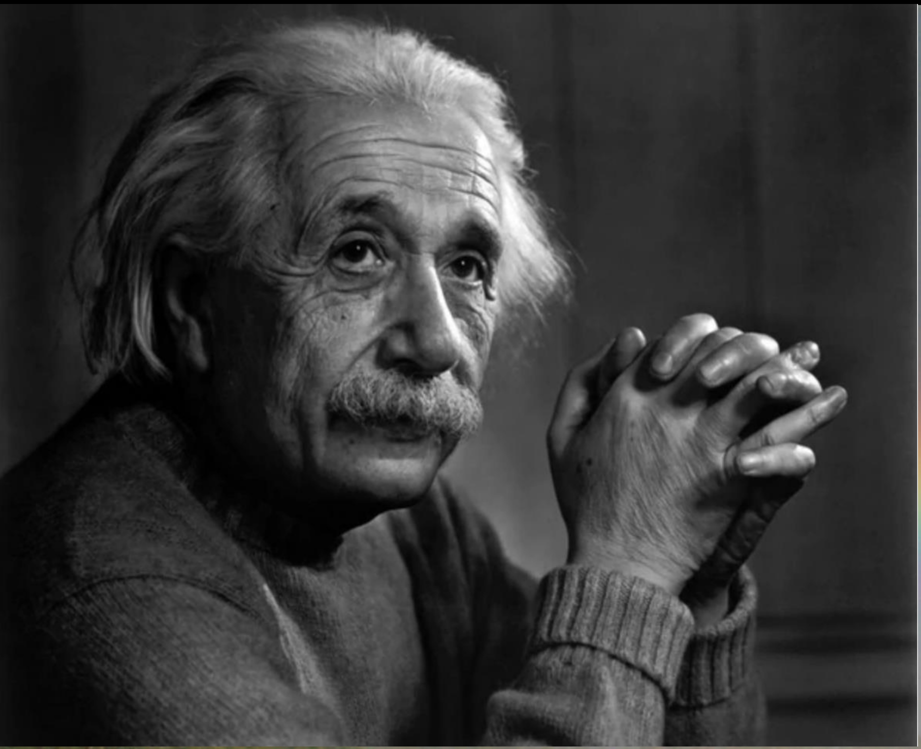
$$\frac{1}{\sqrt{2}} \left(\psi_A \otimes \psi_B + \phi_A \otimes \phi_B \right)$$

Composite systems and entanglement

$$U(t) = e^{-iHt/\hbar}$$

Unitary time-evolution operator

Relax!



"According to the assumption to be contemplated here, when a light ray is spreading from a point, the energy is not distributed continuously over ever-increasing spaces, but consists of a finite number of energy **quanta** that are localized in points in space, move without dividing, and can be absorbed or generated only as a whole."

Annalen der Physik, Band 17, Seite 132-148 (1905)

For more information on mass energy equivalence, see

Annalen der Physik, Band 17, Seite 132-148 (1905)

Original paper

Einstein's four revolutionary 1905 papers see

Einstein's Proposal of the Photon Concept—a Translation of the *Annalen der Physik* Paper of 1905*

A. B. Arons† and M. B. Peppard‡
Amherst College, Amherst, Massachusetts

(Received 2 December 1964)

Of the trio of famous papers that Albert Einstein sent to the *Annalen der Physik* in 1905 only the paper proposing the photon concept has been unavailable in English translation. The *American Journal of Physics* is publishing the following translation in recognition of the sixtieth anniversary of the appearance of the original work. Physics teachers may take particular interest in the following aspects: (1) Einstein's keen awareness of the heuristic character of his new conception. (2) His demonstration from *thermodynamic* and *statistical* considerations that electromagnetic radiation might be conceived as consisting of finite numbers of discrete corpuscles of energy $h\nu$. (3) His *prediction* of the linear relation between the stopping potential of photoelectrons and the frequency of the incident light. This latter aspect of the photoelectric effect was not included among Lenard's early investigations. It remained for Millikan and others to develop the elegant experimental techniques that confirmed Einstein's bold prediction. Readers interested in pursuing the background in greater depth will find it rewarding to refer to the critical analyses by Martin J. Klein in "Einstein's First Paper on Quanta," in *The Natural Philosopher* (Blaisdell Publishing Company, New York, 1963), Vol. II, and "Einstein and the Wave–Particle Duality," in *The Natural Philosopher*, Vol. III, 1964. We are grateful to Professor Klein for his criticism and advice regarding this translation and for his generosity in making available to us an unpublished translation of his own.

CONCERNING AN HEURISTIC POINT OF VIEW TOWARD THE EMISSION AND TRANSFORMATION OF LIGHT

BY A. EINSTEIN

A PROFOUND formal distinction exists between the theoretical concepts which physicists have formed regarding gases and other ponderable bodies and the Maxwellian theory of electromagnetic processes in so-called empty space. While we consider the state of a body to be completely determined by the positions and velocities of a very large, yet finite, number of atoms and electrons, we make use of continuous spatial functions to describe the electromagnetic state of a given volume, and a finite number of parameters cannot be regarded as sufficient for the complete determination of such a state. According to the Maxwellian theory, energy is to be considered a continuous spatial function in the case of all purely electromagnetic phenomena including light, while the energy of a ponderable object should, according to the present conceptions of physicists, be represented as a sum carried over the atoms and electrons. The energy of a ponderable body cannot be subdivided into arbitrarily many or arbitrarily small parts, while the energy of a beam of light from a point source (according to the Maxwellian theory of light or, more generally, according to any wave theory) is

* Ann. Physik 17, 132 (1905); Translation published with the permission of *Annalen der Physik*.
† Department of Physics.
‡ Department of German.

The E = mc^2 paper

← English Translation

↑ German

Doc. 24

DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY CONTENT?

by A. Einstein

[*Annalen der Physik* 18 (1905): 639-641]

E = mc^2 is not mentioned pe se

The results of an electrodynamic investigation published by me recently in this journal[1] lead to a very interesting conclusion, which shall be derived here.

There I based myself upon the Maxwell-Hertz equations for empty space along with Maxwell's expression for the electromagnetic energy of space, and also on the following principle:

The laws governing the changes of state of physical systems do not depend on which one of two coordinate systems moving in uniform parallel translation relative to each other these changes of state are referred to (principle of relativity).

Based on these fundamental principles[2], I derived the following result, among others (*loc. cit.*, §8):

"If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2…the mass of a body is a measure of its energy-content." Substitute E for L you get $m = E/c^2$ or **$E = mc^2$**

New Definition of the kg

The kilogram is **equal to the mass of the energy** of a photon of a specific frequency via Planck's Constant

$E = hf$ (Planck-Einstein equation)

$E = mc^2$

$\therefore mc^2 = hf$

$m = hf \div c^2$

h is Planck's Constant and f is the frequency of a photon
Where E is energy (joules), m is mass (kg), c is the speed of light (m/s)

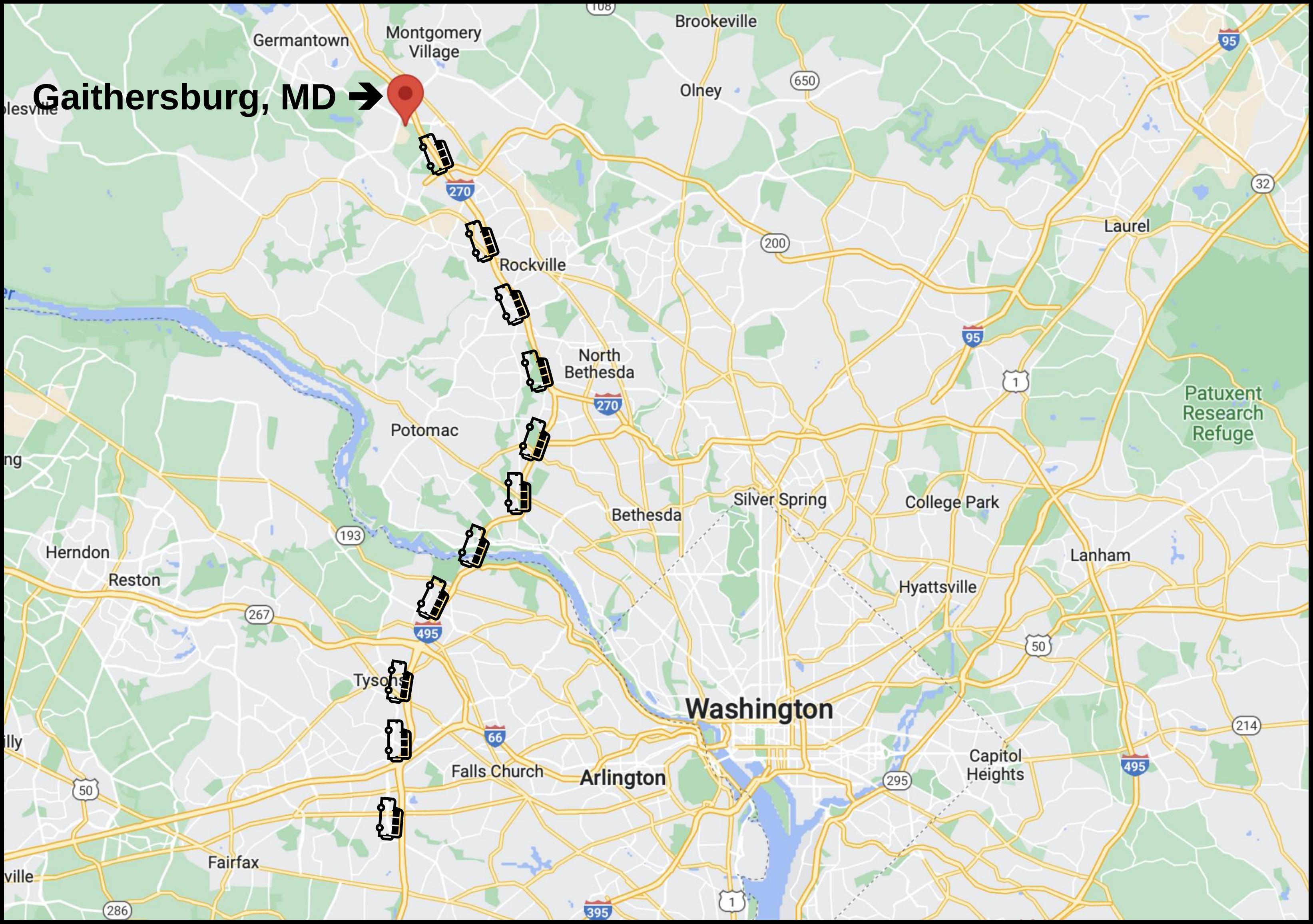
Comparing Visible Light & X-Rays

- Visible light
  - ✓ ~ 6 x 10^{14} hertz (cycles/second)
  - ✓ ~ 2.5 eV energy
- X-Ray radiation
  - ✓ 10^{16} to 10^{20} hertz
  - ✓ 145 to 124,000 eV

] ← **comparing**

What is an eV?

- The amount of kinetic energy gained by a single electron accelerated from rest through an electric potential difference of one volt (in a vacuum)
- 1 eV = 1.602176634 x 10^{-19} Joules
- For more information, see


Gaithersburg, MD ➔
Germantown
Montgomery Village
Brookeville
Olney
Laurel
Rockville
North Bethesda
Potomac
Patuxent Research Refuge
Silver Spring
College Park
Bethesda
Herndon
Reston
Lanham
Hyattsville
Tysons
Washington
Capitol Heights
Falls Church
Arlington
Fairfax
270
495
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267
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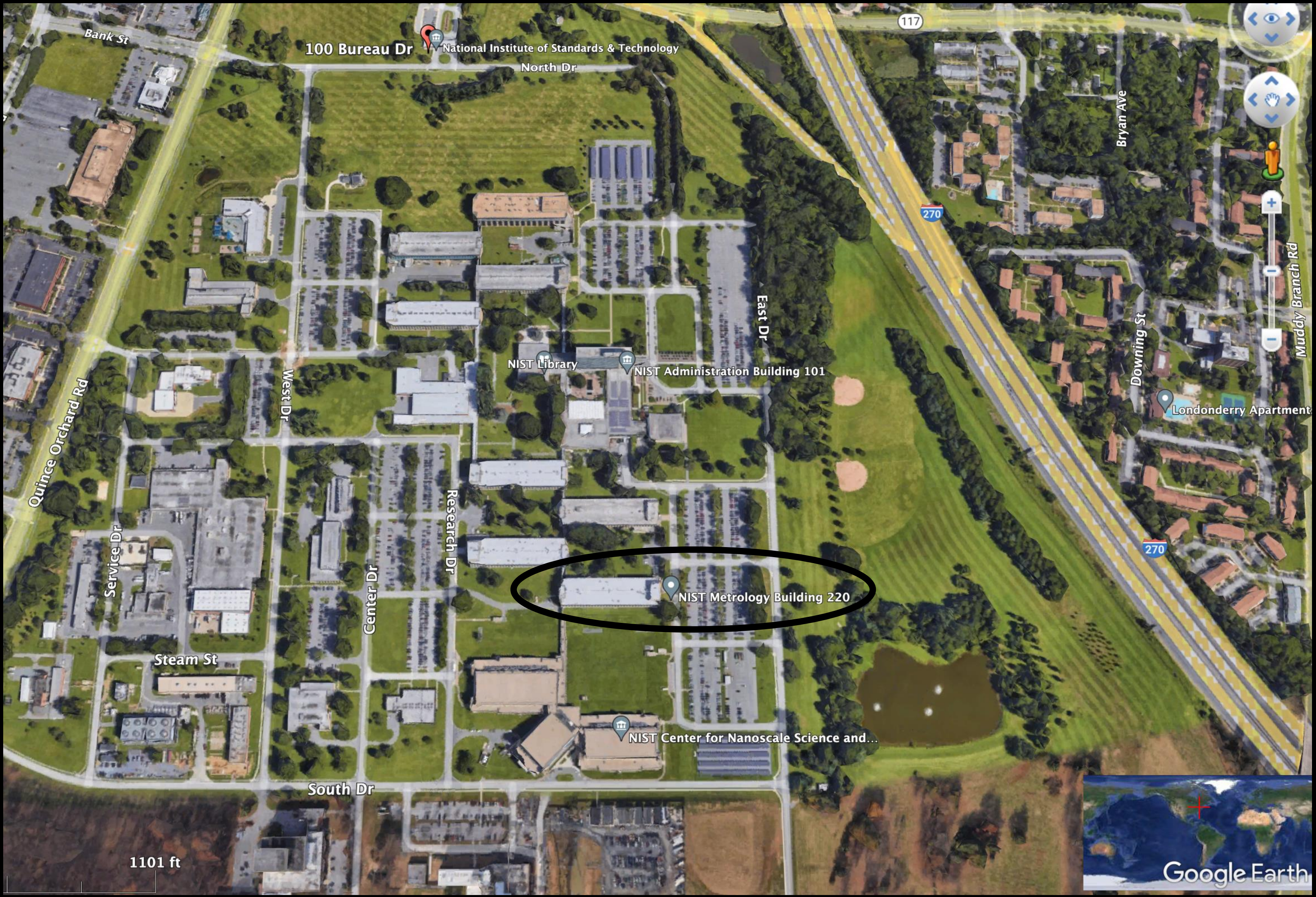
NIST
NATIONAL INSTITUTE
OF STANDARDS AND
TECHNOLOGY
UNITED STATES DEPARTMENT OF COMMERCE
100 Bureau Drive


Highway 270

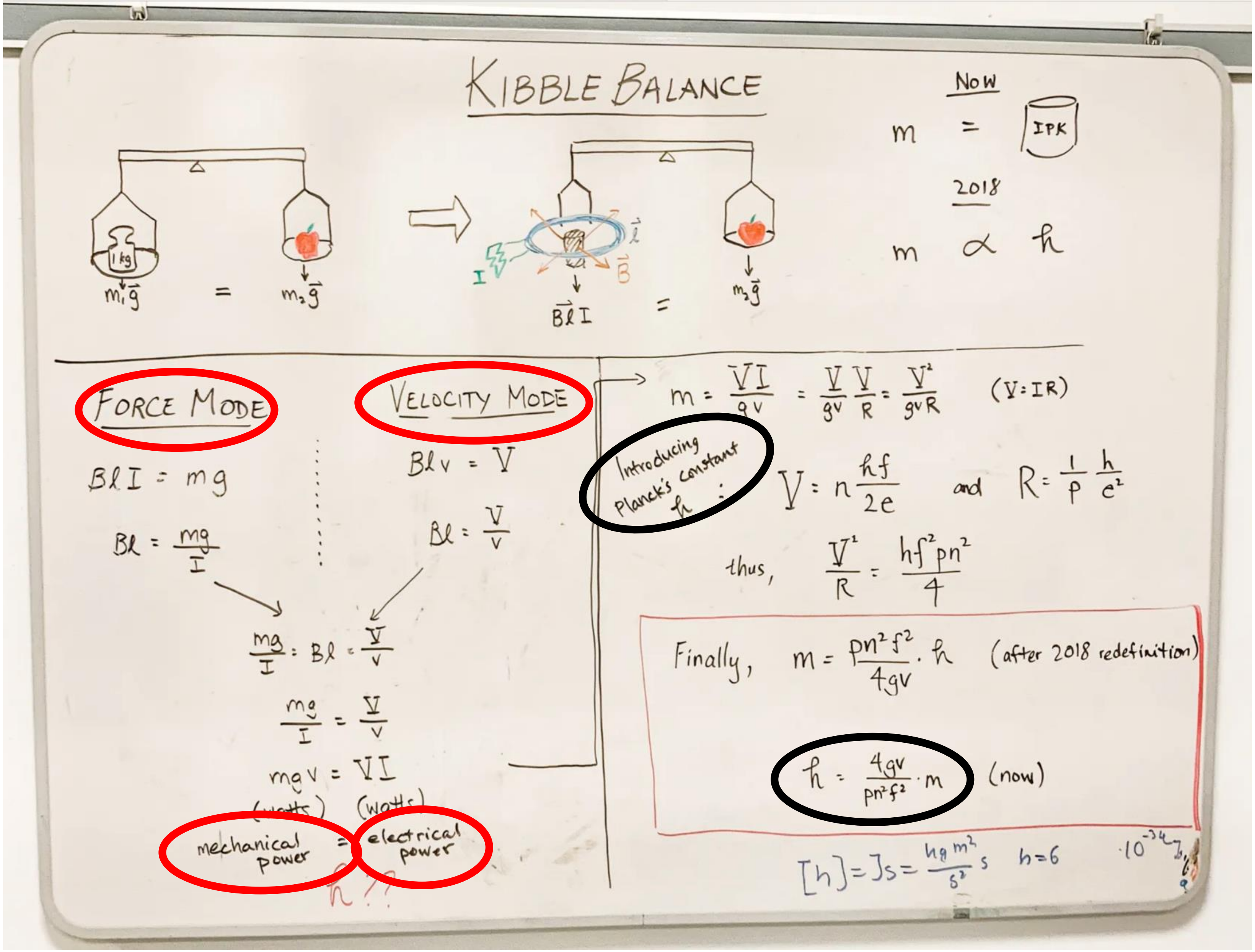
From Google Earth



The Kibble Balance*

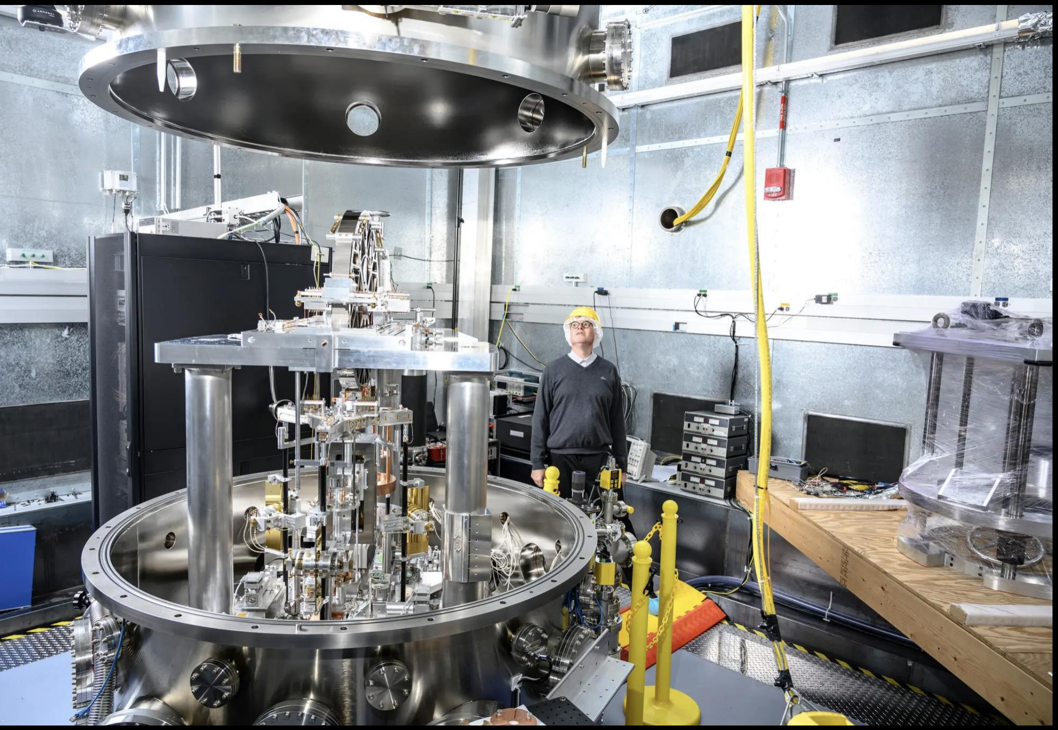
* Originally called the watt balance; named after Bryan Kibble (1938-2016)

Mathematics of the Kibble Balance



To be explained in the forthcoming video in greater detail

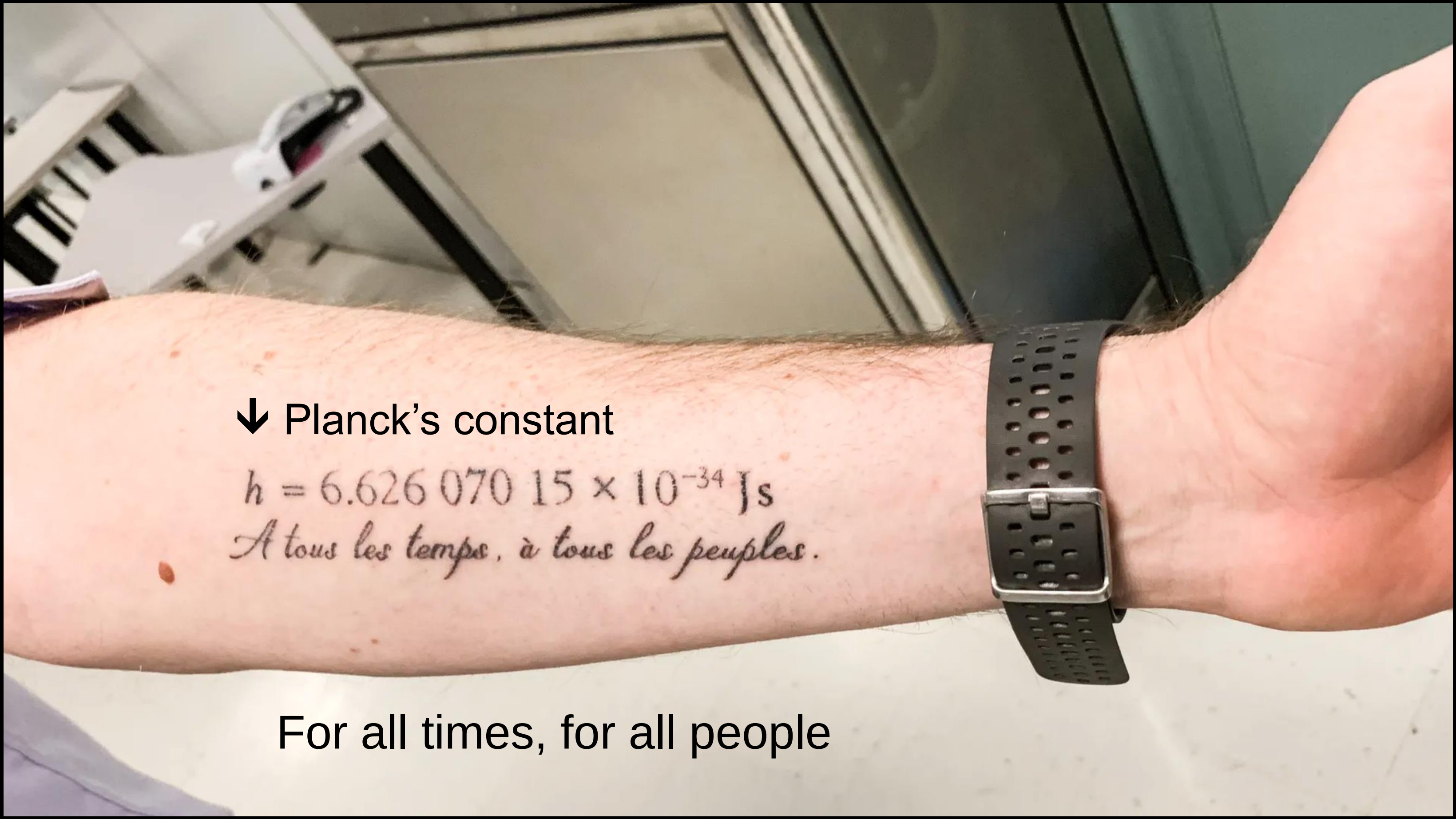
The Kibble Balance—a 10 Minute Video



Posted July 12, 2017

For more information about the Kibble balance, see

Stephan Schlamminger's Tattoo



A scientist at NIST

More Information—Books

- *The New International System of Units (SI) by Ernst O. Göbel and Uwe Siegner*
- *Mass Metrology: The Newly Defined Kilogram* by S.V. Gupta
- *Beyond Measure: The Hidden History of Measurement from Cubits to Quantum Mechanics* by James Vincent
- *Measuring the World: Making Complicated Problems Simple by Really Going Metric* by John Austin

More Information—Videos

- Units of Measure: Scientific Measurements & SI System
- The kg is Dead, Long Live the kg
- Is America Actually Metric?
- Introduction to Metrology…
- Quest for an International Standard Measure…
- What Do You Mean Mass is Energy?
- The Real Meaning of $E = mc^2$
- Planck's Constant and the Origin of Quantum Mechanics
- Boltzmann's Constant
- What is Quantum Mechanics Explained
- A Guide to Imperial Measurements with Matt Parker
- Why Don't We Have Metric Time?

The End

Thanks for your attention!